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Mary Somerville's Early Contributions to the Circulation of Differential Calculus

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Abstract

In 1831, with the publication of her translation of Laplace's *Mécanique Céleste*, Mary Somerville (1780–1872) solidified her reputation throughout Western Europe as a highly proficient mathematician, and an expert in ‘French analysis’. To shed light on her preliminary studies, we here examine Somerville's earliest known mathematical work, namely her published and unpublished solutions to questions posed in the *New Series of the Mathematical Repository*, alongside her contemporary correspondence with mathematicians John and William Wallace. These submissions demonstrate her active engagement in the circulation of the differential calculus twenty years earlier than previously appreciated.

Keywords: Mary Somerville, Differential Calculus, Periodicals, William Wallace, Circulation

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Introduction

Whilst much scholarship has been produced on the circulation of ‘continental’ or ‘French’ mathematics in Britain in the early 19th century, specifically with regards to the calculus,¹ it is only recently that the role of Mary Somerville (1780–1872) has come to be appreciated. Notably in Craik's recent re-examination of the reception of late-eighteenth-century French mathematical analysis in Britain up until 1831, her *Mechanism of the Heavens* (1831) is described as a “masterly exposition of Laplacian astronomy”, and a culmination of 30 years of conscious efforts to embed continental analysis into British mathematics (Craik, 2016, 234), (Somerville, 1831). However, little attention has thus far been given to Somerville's mathematical formation, or indeed her earlier contributions to the circulation of the differential calculus in Great Britain.² As a

¹See (Dubbey, 1963), (Enros, 1983), (Guicciardini, 1989), (Panteki, 1987), (Richards, 1991), (Craik, 1999).

²‘Differential’ is here used to distinguish between the two different styles of calculus present in the early nineteenth century (that is fluxional and differential, see section 3), rather than to suggest a restriction to methods of differentiation.

woman at the beginning of the nineteenth century with little personal means, a consideration of how Somerville enabled herself to study mathematics, and the type of mathematics she chose to pursue, provides a new insight into the accessibility of French analysis at this time in Britain.

Somerville's autobiography, published posthumously in 1873, has heavily influenced the narrative of her early life. This paper therefore begins with a brief overview of Somerville's own account of her early mathematical studies. However, whilst this account is an invaluable source of biographical information, it provides little information on the mathematical resources she had access to at this time, nor her level of engagement with such texts. In order to supplement this account, we begin by considering Somerville's first known publications, in the context of question and answer sections in early-nineteenth-century periodicals. By 1812, Somerville had begun applying the differential calculus in her solutions to questions published in such periodicals; thus Section 3 focuses on the significance of such usage in Britain at that time, its place in the reform of British mathematics, and the ways in which Somerville was able to develop her knowledge of uncommon practice. Section 4 compares early manuscript drafts of her solutions to those published in the *New Series of the Mathematical Repository*, in conjunction with critique by William Wallace with whom she corresponded, to demonstrate Somerville's growing facility with the differential calculus between 1812 and 1816. Finally, although Somerville published under a pseudonym until 1826, by active engagement in polite scientific society throughout Western Europe she nonetheless cultivated a reputation for herself as an accomplished mathematician. Consequently Section 5 considers contemporary letters to illuminate her reception in such social spaces, focusing on accounts which discuss her mathematical accomplishments.

1. An Autobiographical Account

I became acquainted with Mr. Wallace, who was, if I am not mistaken, mathematical teacher of the Military College at Marlow, and editor of a mathematical journal published there. I had solved some of the problems contained in it and sent them to him, which led to a correspondence, as Mr. Wallace sent me his own solutions in return.

(Somerville and Somerville, 1873, 78–9)

In the late 1860s, Mary Somerville began preparing an autobiography, to be published posthumously, which is now an invaluable source of information on her life. Having assembled a substantial collection of letters, notebooks and manuscripts to which she could refer, Somerville was able to give a detailed account of her life at the centre of polite scientific society throughout much of the nineteenth century.³ Notably, we get a first-hand account of the life of a woman

³We here use 'polite' to indicate the social prestige of the communities with which

who chose to pursue her mathematical and scientific interests, authoring papers in the *Philosophical Transactions* of the Royal Society of London and three highly acclaimed books, at a time when the contributions of women were, or have since been rendered, invisible. Since its publication in 1873, with editorial notes by her daughter Martha Charters Somerville, *The Personal Recollections, from Early Life to Old Age, of Mary Somerville* (PR) has dominated the narrative around Somerville’s early life, and most accounts rely on it almost completely (for example those found in (Chapman, 2015), (Neeley, 2001) and (Patterson, 1983)). We begin with an overview of Somerville’s account of her early life and introduction to mathematics.⁴

The eldest child of Vice-Admiral Sir William George Fairfax and his second wife Margaret Charters, Mary Fairfax⁵ was born on 26th September 1780. Her love of nature was sparked by explorations of the beaches by her family home in Burntisland, near Edinburgh, whilst her formal education was apparently limited to reading the Bible, studies of “the common rules of arithmetic” at a writing school, a single year at a boarding school, and attendance at a village school for needlework (Somerville and Somerville, 1873, 36). In addition, in the hopes of reducing her “strong Scotch accent”, Somerville’s father made her read aloud a paper a day from *The Spectator*, a periodical published almost daily between March 1711 and December 1712 (Somerville and Somerville, 1873, 20).⁶

Somerville’s discovery of algebra occurred when reading a ladies magazine with a certain ‘Miss Ogilvie’, who described the subject as “a type of arithmetic” (Somerville and Somerville, 1873, 47). She was initially unable to find any further information regarding algebra, as none of her immediate family had an interest in such things, nor would she have had the courage to ask them if they had for fear she “should have been laughed at”; Somerville described herself at this time as “often very sad and forlorn; not a hand held out to help me” (Somerville and Somerville, 1873, 48). Later, through her brother’s tutor, she was able to acquire copies of Euclid’s *Elements* and Bonnycastle’s *Algebra*, which she claimed were the books used in schools at that time.⁷ She proceeded

Somerville was engaging, usually the landed gentry or equivalent. See (Vickery, 1999, 13).

⁴As indicated in the full title, PR gives an account of Somerville’s life from her youth to her death. In this paper we will focus solely on the passages treating Somerville’s life up to the 1820s, but it repays a complete reading owing to its insightful and emotive description of Somerville’s love of science and nature, and her life in a nineteenth-century scientific community.

⁵I will refer to her as Somerville throughout this paper for ease, as that is the name under which she published from 1826 onwards, and was known in the scientific community.

⁶The articles contained in *The Spectator* were republished in seven bound volumes between 1712 and 1713; these volumes were reprinted often throughout the eighteenth century in at least London, Edinburgh and Dublin and played a notable role in Scottish Enlightenment thought (Bond et al., 1965, Vol 1, v), (Phillipson, 1981, 26), (Broadie, 2003). This iteration of *The Spectator* is unrelated to the weekly magazine which bears the same name nowadays.

⁷More than five English editions of Euclid’s *Elements* were published in Britain in the eighteenth century; Robert Simson’s 1756 *The Elements of Euclid* became a key mathematical textbook in Scotland, both in schools and universities and was reprinted 26 times by 1780 (Barrow-Green, 2006, 10–3), (Ackerberg-Hastings, 2002, 48). John Bonnycastle, a mathemat-

to study these books independently at night—even after her candles were confiscated by her parents, who were most displeased at their daughter’s night-time activities. Discouraged by her family members, Somerville recollected; “I felt in my own breast that women were capable of taking a higher place in creation than that assigned to them in my early days, which was very low” (Somerville and Somerville, 1873, 60). Undeterred, she would rise at day-break, wrap herself in a blanket and “read algebra or the classics till breakfast time” (Somerville and Somerville, 1873, 65). Somerville’s isolation increased further on her marriage to her second cousin Samuel Greig (1778–1807) in 1804, with whom she moved to London. He passed away after only three years of marriage, and subsequently Somerville returned to her family home in Burntisland, a widow and mother of two sons with limited independent means. She there resumed her mathematical studies in earnest, and after studying “plane and spherical trigonometry, conic sections and... astronomy”, Somerville turned to Isaac Newton’s *Principia*, which she found “extremely difficult” on first reading (Somerville and Somerville, 1873, 78).

In a time when social standing and rank greatly determined one’s prospects, Somerville benefited significantly from her place amongst the minor gentry. Through her mother, Somerville was distantly related to the Earl of Minto, and her father claimed to be connected to the Barons Fairfax of Cameron in the Scottish peerage (Somerville and Somerville, 1873, 6–8). Although her immediate family were not themselves notably wealthy or amongst the peerage,⁸ her father was knighted in recognition of his part in the 1797 Battle of Camperdown and was thus entitled to the prefix ‘Sir’. As a physician Somerville’s second husband William Somerville (1771–1860) would have ranked amongst baronets and knights, and his family was thus entitled to be presented at the Queen’s Drawing Room in St James’ Palace; indeed in 1837 Somerville attended the coronation of Queen Victoria (Somerville and Somerville, 1873, 148, 199).

By Somerville’s account, she was welcomed into Edinburgh polite society from a young age, often sitting with ladies in their boxes at the theatre, and attending both public and private balls;⁹ in preparation for these she had attended “Strange’s dancing school”, where she learnt reels and country dances whilst in full evening dress (Somerville and Somerville, 1873, 43, 52). Whilst out in Edinburgh society she became acquainted with a “small society of men of the most liberal principles” who conducted the *Edinburgh Review*. Somerville specifically mentions Sydney Smith, a well-known author and moral philosopher, Henry Brougham, a lawyer (and later Baron Brougham and Vaux), and John Playfair, who held the chair of natural philosophy at the University of Ed-

ical master at the Royal Military Academy in Woolwich, first published his *An Introduction to Algebra* in 1782, and by 1824 it had reached its thirteenth edition (Whittaker and Rice, 2004), (Bonycastle, 1782).

⁸Dukes, Marquesses, Earls, Viscounts, and Barons made up the peerage, whose titles were almost always hereditary.

⁹She supposedly here earned the nickname the ‘Rose of Jedwood’, which references both her beauty and her birth place of Jedburgh (Somerville and Somerville, 1873, 61).

inburgh (Somerville and Somerville, 1873, 63–5, 81–3).¹⁰ Playfair would later nominate William Somerville for membership of the Royal Society of Edinburgh, through which William and Mary Somerville became more closely acquainted with those interested in the sciences, both in Edinburgh and London.¹¹ After a brief mention of her solutions in the *Mathematical Repository*, and advice given to her by Playfair for reading Pierre-Simon Laplace’s *Traité de Mécanique Céleste* (Laplace, 1799), mathematics receives little more attention in PR until the narrative reaches the 1820s.

The account given in PR provides no explanation as to how, by 1825, Somerville was able to transition from an isolated amateur, with little access to mathematical texts, to a published mathematician recognised throughout Great Britain and Western Europe. Although Somerville described her expanding social network, which by this time included scientists and mathematicians based in at least Edinburgh, London, Paris, Brussels, and Geneva, no work has yet been done to investigate which mathematics Somerville was engaging with at this time, nor how. Moreover, it is clear throughout PR that Somerville was aware of her value as a symbol of scientific attainment in the ‘fairer sex’, as well as of the self-improvement which can be gained through the independent pursuit of knowledge (Somerville and Secord, 2004, xi–xii, Vol 9). PR thus presents a heavily curated view of her life, deeply shaped by the materials she had to hand and her desire to advocate for higher education for women.¹²

Therefore in this paper we will turn to contemporary letters and notebooks to supplement Somerville’s autobiographical account and present an expanded time-line of her early engagement with mathematics.

¹⁰For more biographical information on Smith, Brougham and Playfair see the *Oxford Dictionary of National Biography*.

¹¹For more information on how William influenced and assisted Mary Somerville’s intellectual pursuits, see the forthcoming article by the current author.

¹²Inconsistencies and inaccuracies were introduced when Somerville relied on memory alone, having no letter or notebook to which to refer. For example she claims that whilst Augustus De Morgan adopted the differential calculus, “several years elapsed before Mr. Herschel and Mr. Babbage were joint-editors with Professor Peacock in publishing an abridged translation of La Croix’s *Treatise on the Differential and Integral Calculus*” (Somerville and Somerville, 1873, 78); but this translation was in fact published in 1816 when De Morgan was only 10 years of age (Lacroix, 1816).

2. A Medal for Mathematics

I had now read a good deal on the higher branches of mathematics and physical astronomy, but as I never had been taught, I was afraid that I might imagine that I understood the subjects when I really did not; so by Professor [William] Wallace's advice I engaged his brother to read with me... Mr. John Wallace was a good mathematician, but I soon found that I understood the subject as well as he did. I was glad, however, to have taken this resolution, as it gave me confidence in myself and consequently courage to persevere.

(Somerville and Somerville, 1873, 82)

The first evidence we have of Somerville publishing mathematics is in a letter written to her by fellow Scot, John Wallace, in July 1811. Wallace is only briefly mentioned by Somerville in PR (namely in the above passage) and little seems to be known of him. He has no entry in the *Dictionary of National Biography* nor the *Dictionary of Scientific Biography*, nor does he appear in the entries of his older brother William Wallace (who will be discussed in more depth in Section 3). The records of the Royal Military College¹³ show that John Wallace was hired as the Master of Arithmetic there in September 1817, aged 36, and remained in post until 1823 when he succeeded to the ministry of a Scottish Parish.¹⁴

John Wallace begins his letter by apologising to Somerville for failing to reply to her last communication, but he is “confident that all excuses are unnecessary” as he has “the pleasure of informing [Somerville] that [her] solution of the prize-question for the Mathematical Repository has gained the prize”.¹⁵

The ‘Mathematical Repository’ mentioned here was in fact the *New Series of the Mathematical Repository* (MR). Edited by Thomas Leybourn, master of mathematics at the Royal Military College where John Wallace would later work, MR was published in six volumes at irregular intervals between 1806 and 1835 (Guicciardini, 2004). Each volume was divided into three parts: one of ‘Original Essays on Mathematical Subjects’; one of ‘Mathematical Memoirs, extracted from Works of Eminence’; and finally questions ‘in almost every branch of mathematics’ together with their solutions as submitted by readers (Leybourn, 1806–1835, Advertisement, Vol 1). In 1814, from the third volume onwards, a fourth part was introduced entitled ‘Cambridge Problems’, in which the Senate-House questions¹⁶ “given to the Candidates for Honours during the

¹³Based in Great Marlow, near London, and subsequently in Sandhurst from 1812 until its closure in 1939. In 1947 the Royal Military Academy was subsequently founded on the same site in Sandhurst.

¹⁴The Sandhurst Collection, Image ID: 8650pge00142, www.sandhurstcollection.co.uk.

¹⁵Mary Somerville Collection, Bodleian Library, Oxford (MSC), Dep. c. 375, Folder MSDIP–1, 12/07/1811.

¹⁶The Senate House exam is often also known as the Mathematical Tripos; for more in-

Examination for the Degree of B. A.” at the University of Cambridge were reproduced (Leybourn, 1806–1835, 1, Section 4, Vol 3).

The question and answer section in each volume contained up to 120 questions, separated into 4 numbers; the final question in each number was designated a ‘Prize Question’, for the best solution of which the editors of MR would award a specially cast silver medal.¹⁷ The Prize Questions do not differ tangibly from the other problems included in MR, either by content or difficulty. Both were submitted by a variety of contributors, from professors of mathematics at the Royal Military College to provincial gentlemen,¹⁸ with many submitted under pseudonyms. One puzzle was extracted from a memoir of Gauss (Prize Puzzle 490, Volume 5), and solutions to at least two questions (Prize Puzzle 390, Volume 4 and 430, Volume 5) were extracted from the *Annales de Mathématiques pures et appliquées*.¹⁹

Mathematical periodicals with Q&A sections, such as MR, played a key role in the education and careers of those whose means precluded them from school or university, as demonstrated in the life of contributor John Butterworth, an autodidact who went on to supplement his income by solving mathematical puzzles for others and ultimately opened a school (Despeaux, 2014, 17). Other contributors built a name for themselves through submitting their solutions to periodicals, and then went on to become staff members at the Royal Military College itself. These included James Cunliffe, who we will meet again later, who submitted questions and solutions to both MR and the *Ladies’ Diary* before being hired in March 1805 as a Master of Arithmetic, and was then promoted in 1819 to a Professor of Mathematics,²⁰ and of course John Wallace himself was later hired by the college.

At this time, the posing and answering of mathematical puzzles in such periodicals was consciously seen by practitioners as a way to actively contribute to mathematical knowledge. Q&A sections were included in periodicals and

formation on the examination system at Cambridge see (Warwick, 2003, 52–8). Collections of the Senate-House questions were also published in separate volumes, for example (Wright, 1825) and (Anon, 1837).

¹⁷The practice of publishing mathematical questions and answers submitted by readers in almanacs and periodicals can be traced back to the beginning of the eighteenth century; Despeaux has identified almost forty works which contain such sections published in Britain during the eighteenth and nineteenth centuries (Despeaux, 2014, 55). Of these, the *Ladies’ Diary* is perhaps most well known. Founded in 1704, it ran for 136 years and contained mathematical questions alongside word puzzles, calendars with notable dates, and lists of upcoming eclipses. Puzzles were both submitted and answered by women, and when retrospectively categorised in 1817 the questions covered topics such as algebra, geometry, fluxions, hydrostatics, optics and more (Perl, 1979, 37–9). See also (Costa, 2002) and (Albree and Brown, 2009).

¹⁸Such as a Mr Mason of Scoulton, “a gentleman whose labours... enriched the English periodicals for several years” (Anon, 1836, 25). Submissions for the Q&A section came from readers across Great Britain and Ireland, including Plymouth, London, Birmingham, Liverpool, Bolton, Edinburgh, Dublin, and Carlow.

¹⁹A mathematical journal edited by Joseph Diez Gergonne (1771–1859) and published in Nîmes, France from 1810–1831. The journal is commonly known as Gergonne’s *Annales*.

²⁰The Sandhurst Collection, Image ID: 8529pge00021, www.sandhurstcollection.co.uk.

journals throughout Western Europe in the eighteenth and nineteenth centuries and, as we will see in the case of MR, contained highly advanced and innovative mathematics (Despeaux, 2014, 47–50). According to the Advertisement of the first volume of MR

The utility of this part of the work [the Q&A section] will be readily admitted when it is considered, that almost all the improvements which the Mathematics have received, have originated in the exertions made to resolve particular problems, such as that of the *trisection of an angle* among the ancients; also the various *isoperimetrical problems*, and above all, the problem of *the three bodies* among the moderns. We believe also, that most Mathematicians will confess how much their talents have been cultivated and their knowledge improved, by resolving problems, such as are proposed in this volume (Leybourn, 1806–1835, Advertisement, Vol 1).

Thus when Somerville chose to submit her solution to the *New Series of the Mathematical Repository*, she was engaging in a well-established and highly valued mathematical practice, and moving from a passive consumer of knowledge to an active contributor.

John Wallace’s delight and pride in Somerville being awarded a silver medal for her MR submission are evident in his aforementioned letter.²¹ His subsequent description of her as his “pupil” in the same letter indicates he had a much more formative influence on her mathematical studies than Somerville’s account of him as a mere reading companion would otherwise suggest. It is very unlikely that Wallace is here implying Somerville paid him for private tutoring. More probable is that they became acquainted in Edinburgh society, and on discovering a shared interest in mathematics pursued a closer acquaintance which developed into that of informal mentor and mentee.²²

However, although John Wallace announced to Somerville that her solution had been selected for a prize, and we still have the medal which Somerville was awarded (see Figure 1), no contributions appear in any volume of this periodical under the name Mary Fairfax, Greig or Somerville. Just over a month before Wallace’s letter, on June 1 1811, submissions had closed for solutions to Questions 291–310, which were subsequently published in Volume 3 of MR in 1814 (Leybourn, 1806–1835, Vol 3). A handwritten copy of the winning solution to Prize Question 310 held in the Somerville Collection,²³ suggests that Somerville’s contributions were published under the pseudonym “a Lady” (Somerville and Secord, 2004, xlv, Vol 1).

As mentioned above, solutions were often published under pseudonyms, with some authors publishing under multiple identities as well as their own name. It is

²¹MSC, Dep. c. 375, Folder MSDIP–1, 12/07/1811.

²²We see this process again later in Somerville’s life through her informal tutoring of Ada Lovelace (Hollings et al., 2017, 231).

²³MSC Dep. c. 372, Folder MSW–1; see Appendix A.



Figure 1: The medal awarded to Mary Somerville for her solution of Prize Question 310, posed in Volume 3 of MR. The medal is now held at Somerville College, Oxford and the inscription reads: Maria Greig, L.M.D; PALMAM QUI MERUIT FERAT; T. Leybourn. The Latin loosely translates as ‘let they who have earned the palm, bear it’, where the palm signifies victory; the significance of L.M.D is unclear. Somerville’s name appears in Latin as Maria Greig, as she had not yet married for the second time and assumed the name Somerville.

currently unknown why Somerville’s solutions were published anonymously, but it is clear that they were not *submitted* under a pseudonym; John’s letter implies she submitted her solution via his own brother William, who worked alongside the editor of MR at the RMC.²⁴ In addition, her medal from the editors clearly bears her name, so Somerville’s identity was by no means a well-kept secret.

Both Somerville and John Wallace continued submitting solutions to mathematical puzzles; (Somerville and Secord, 2004) suggests that it is unlikely multiple people would have shared the same pseudonym, and we will see in subsequent sections manuscript evidence that all five solutions published in MR under the pseudonym “a Lady” were in fact written by Somerville. In addition, she prepared solutions to a further three questions, one of which provides the first written record of her usage of the differential calculus.

²⁴John wrote on 12th July 1811 that his brother had recently arrived in Edinburgh having set off from Marlow thirteen days previously, just after receiving Somerville’s solution. (MSC Dep. c. 375, Folder MSDIP-1, 12/07/1811).

3. Studying the Calculus

To trace the history of the differential calculus through the cloud of dispute and national acrimony, which has been thrown over its origin, would answer little purpose. It is a lamentable consideration, that that discovery which has most of any done honour to the genius of man, should nevertheless bring with it a train of reflections so little to the credit of his heart.

Memoirs of the Analytical Society (Anon, 1813, iv)

The almost simultaneous development in the seventeenth century of two different formalisms of the calculus, by Isaac Newton and Gottfried Wilhelm Leibniz, and the subsequent priority dispute have been well treated. Owing to this dispute, alongside many other cultural and intellectual factors, mathematical practice in Great Britain became perceivably different to that in continental Europe, as adherence to the different formalisms had “suggest[ed] separate directions for research and therefore generate[d] different kinds of knowledge” (Sigurdsson, 1992, 110). Pertinently to this essay, the differing practices visibly manifested in the mathematical notation used; the fluxional notation of Newton, \dot{x}, \dot{y} , was most prevalent in the work of British mathematicians, and the differential notation introduced by Leibniz, dy, dx , was referred to as the ‘foreign notation’.

By the beginning of the nineteenth century this incongruity of practice was interpreted by some as a mark of the inferiority of British mathematics. In the preface to the first memoir of the Analytical Society, founded in 1812 by a small group of students at Cambridge University, it was claimed that differential calculus had “dropped and almost faded into neglect” in Britain, and thus it was now necessary “to re-import the exotic, with nearly a century of foreign improvement” (Anon, 1813, iv).²⁵ Somerville herself commented in PR that “at this period mathematical science was at a low ebb in Britain; reverence for Newton had prevented men from adopting the ‘Calculus’, which had enabled foreign mathematicians to carry astronomical and mechanical science to the highest perfection” (Somerville and Somerville, 1873, 78). In response, French mathematical ideas were consciously circulated through reviews, such as those of John Playfair in the *Edinburgh Review*, and translations, such as the 1816 translation of Sylvestre-François Lacroix’s *Traité Élémentaire de calcul différentiel et de calcul intégral* by the Analytical Society (Ackerberg-Hastings, 2008), (Lacroix, 1816).²⁶ But this mathematics was not uniformly welcomed in Britain; George Peacock, a founding member of the Analytical Society who went on to become a mathematics lecturer and later Lowndean Professor of Astronomy and Geometry in Cambridge, generated outcry in 1817 when he introduced differential

²⁵For more information on the Analytical Society see (Enros, 1983).

²⁶For more on the circulation of the differential calculus in Great Britain at the beginning of the nineteenth-century see (Craik, 2016) and (Guicciardini, 1989).

notation into the Senate House Exams (Warwick, 2003, 68).

The *New Series of the Mathematical Repository* provided its readers with access to continental mathematics through a series of ‘notices relating to mathematics’ which were consistently international in their outlook. For example, Volume 1 contained an announcement of the Prize Question of the Institut de France regarding the “Theory of the Perturbations of the Planet Pallas”, whilst Volume 3 listed the authors and titles (translated into English) of all mathematical papers contained in the first 15 numbers of the *Journal d’École Polytechnique*. There were also announcements of recently published ‘foreign books’ (predominantly published in French), and later volumes contained obituaries of mathematicians such as Joseph-Louis Lagrange, Pierre-Simon Laplace and Gaspard Monge. The section of MR titled ‘Works of Eminence’ featured an anonymous translation of a 1798 work on ‘spherical triangles’ by Lagrange, and of a 1794 ‘memoir on elliptic transcendents’ by Adrien-Marie Legendre (Lagrange, 1798) and (Legendre, 1794); William Wallace is identified as the translator of both these works in (Panteki, 1987, 121).

Some of the first examples of differential notation as used by mathematicians working in Britain can be found in the Q&A section of MR. As such, it was described as “one of the most important works in the reform of the British Calculus” in (Guicciardini, 1989, 116). As early as 1809, Volume 2 contained four solutions which utilised differential notation, three of which were submitted by James Ivory, a Professor at the Royal Military College in Marlow.²⁷

The fourth solution was submitted by William Wallace, the older brother of Somerville’s tutor, John Wallace. Similarly to Somerville, William’s mathematical studies had begun later in his life; as a bookbinder’s apprentice in Edinburgh he pursued learning independently, before attending the lectures of John Robison at Edinburgh University. Through Robison he was introduced to John Playfair, who in 1794 recommended him for the position of mathematical teacher at Perth Academy.²⁸ William Wallace moved to the Royal Military College, Marlow in 1803, where he worked alongside James Ivory and, from 1817, his brother John.²⁹ In 1819, William Wallace left the RMC to take up the chair in mathematics at Edinburgh University, where he remained until he retired from ill health in 1838 (Stronach and Panteki, 2004). His work on the differential calculus, and his translations of French mathematics are well treated in (Guicciardini, 1989), (Panteki, 1987), and (Craik, 1999).

William Wallace himself certainly saw his adoption of differential notation as

²⁷See Questions 151, 160, 172 (Leybourn, 1806–1835, 65–124, Vol 2), as identified in (Panteki, 1987, 123). Ivory was later recognised throughout Europe for his work on the attractions of ellipsoids, published in the *Philosophical Transactions* of the Royal Society in 1809 (Craik, 2016, 250), (Anderson and Grattan-Guinness, 2005).

²⁸It is possible that Somerville became acquainted with the Wallace brothers through Playfair, with whom she also discussed mathematics, but this is not confirmed in PR nor in any correspondence seen by this author.

²⁹For more information on the role of the Royal Military College in the circulation of the differential calculus see (Guicciardini, 1989, 114–5).

an act of reform. In c.1834 he penned a letter to George Peacock, in response to the latter's *Report on Certain Branches of Analysis* (Peacock, 1834), in which Wallace objected that he had left out notable contributions to reform which had been made outside of "Cambridge, the Holy City of Mathematics" (Panteki, 1987, 123–4). Wallace specifically notes his own aforementioned translation of Legendre, and puzzle solutions published in MR in which he employed the "foreign notation" in a "revolutionary spirit" (Panteki, 1987, 123–4). Wallace reiterates the importance of his usage of this notation when writing to Henry Brougham in May of 1835 to request support for his petition for a pension from the British government. Amongst a list of his achievements, including his contributions to encyclopaedias and the invention of mathematical instruments, he explicitly notes that he and James Ivory, "were the first to introduce the Notation of the Continent into Britain in our writings" (Craik, 1999, 262–3).

Somerville's first solution which used the differential calculus is contained in a letter written by her to William Wallace, in April of 1812.³⁰ The highly formal tone of this letter, written in third person, suggests that Somerville and William Wallace were still not yet personally acquainted. Furthermore, Somerville begins the letter by thanking Wallace for the "handsome manner in which he interested himself" in her medal-winning solution, when he facilitated its publication nine months earlier, so this could perhaps have been only their second interaction (Somerville, 1812). Somerville enclosed in the letter her solutions to three questions posed in Volume 3 of MR. Two of these were later included or given an honourable mention in Volume 3 of MR under the pseudonym "a Lady", alongside her prize winning submission; namely Question 317 which treated a construction in Euclidean Geometry, and Question 311 which presented a problem in number theory and was solved using basic algebraic manipulation (see Appendix A).

It is the third solution enclosed with the letter which provides our first view of Somerville using the differential calculus, as applied to the following question, submitted by John Lowry:³¹

XIV. Question 324,³² by Mr. Lowry

With what radius must a circle be described, from a given point as a centre, so that intersecting another circle given by position, the length of the arch [sic] intercepted by the given circle may be a maximum?

³⁰This letter is part of a private collection of manuscripts once belonging to William Wallace, which was brought to the attention of John O'Connor and Alex Craik in 2011. Copies of those items which were deemed to have mathematical interest were subsequently made available via MacTutor (Craik and O'Connor, 2011, 17).

³¹John Lowry was also a Master of Arithmetic at the Royal Military College, Marlow (Platts and Tompson, 2004).

³²The double numbering system in the Q&A sections is used throughout MR; the Roman numeral signifies that this is the fourteenth question in the specific Q&A section, whilst the Indo-Arabic numeral signifies it is the 384th question published in MR.

In order to answer this question, Somerville deduced an expression for the arclength in terms of the radius of the circle given by position, and the length of the chord which begins at a point of intersection of the two circles and meets the line connecting their centres at right-angles. This expression is given in the form of an integral, which Somerville argued must be a maximum. She then applied a variational method and found that this chord is in fact a diameter. Unfortunately a page of the letter is missing, so Somerville’s solution is incomplete. What remains demonstrates Somerville’s adherence to differential notation, alongside some conceptual misunderstandings of the question (Craik and O’Connor, 2011, 21).

It is unclear where or how Somerville would have studied the calculus of variations, but this solution certainly suggests that she had access to advanced mathematical texts before corresponding with William Wallace, perhaps through his brother John Wallace, or John Playfair. A possible text for Somerville to have read is Robert Woodhouse’s 1810 *Treatise on Isoperimetrical Problems and the Calculus of Variations*, which would have been recently printed in Cambridge. Woodhouse claimed that his treatise brought together for the first time disparate results in the study of maxima and minima, or the “calcul des variations”, from both British and continental authors, and rendered them understandable to a modern reader (Woodhouse, 1810, i–iv). However Craik notes that the work “addressed advanced analytical topics and so [was] at first read by few” (Craik, 2016, 245–6). In PR Somerville claims to have purchased Euler’s fundamental book on maxima and minima (Euler, 1744)³³ but not until after corresponding with William Wallace; this text would also have been insufficient on its own, as the δ -notation used by Somerville in her solution was not introduced until 1762 by Lagrange (and subsequently adopted by Woodhouse amongst many others) (Lagrange, 1761–2) (Fraser, 2003, 361).

Although nearly half of the 90 questions included in Volume 3 were answered using calculus of some sort, only thirteen solutions used a form of differential notation, and even then it was often intermingled with fluxional language. Furthermore, eight of those solutions were submitted by William Wallace himself.³⁴ Thus it was perhaps quite a surprise to receive a letter containing this style of mathematics, and from a woman no less. Although there is evidence that John Wallace was also interested in adopting differential notation, namely his solution to Question 266 in Volume 3 of MR, it could have been at this point that Somerville felt she had outgrown his tutelage, as claimed in PR.

Certainly Somerville and William Wallace became much more closely acquainted almost immediately after this letter; whilst travelling to Portsmouth

³³Named by Somerville as Euler’s *Isoperimetrical Problems* written in Latin, see below.

³⁴William Wallace (under various pseudonyms) used differential notation in his solutions to Questions 263, 271, 279, 290, 298, 301, 306, 330. The other solutions which used differential calculus in volume 3 were submitted by: Reverend John Toplis to Question 252; Mr J Wallace, Edinburgh to Question 266 (this could have been John Wallace); Z’s solution to Question 270; Messers Kyn and Williams to Question 297; and A.B.’s solution to Question 300 (Leybourn, 1806–1835, Vol 3).

as a newly-wed in July of 1812 Somerville visited Wallace at the Royal Military College (see Section 5). Furthermore, Somerville suggests in PR it was to William Wallace that she turned for advice when beginning her private collection of mathematical books. He supposedly provided a list of works, mostly in French, for Somerville to read in order to fulfil her intention of following “a regular course of mathematical and astronomical science, even including the highest branches”; she specifically notes “La Croix’s *Algebra* and his large work on the Differential and Integral Calculus, together with his work on Finite Differences and Series, Biot’s *Analytical Geometry and Astronomy*, Poisson’s *Treatise on Mechanics*, La Grange’s *Theory of Analytical Functions*, Euler’s *Algebra*, Euler’s *Isoperimetrical Problems* (in Latin), Clairault’s [sic] *Figure of the Earth*, Monge’s *Application of Analysis to Geometry*, La Place’s *Mécanique Céleste*, and his *Analytical Theory of Probabilities &c., &c., &c....*” (Somerville and Somerville, 1873, 79). Many of the works mentioned here as being part of Somerville’s original collection (to which she added substantially over the following sixty years of her life) are still held together at Girton College, originally a higher education institute for women which is now a mixed college at the University of Cambridge. It is unclear how Somerville was able to purchase copies of these books, but from an inscription in her copy of Sylvestre-François Lacroix’s *Traité du calcul différentiel et du calcul intégral* it appears she began purchasing the books as early as October 1812. Certainly some of the texts were gifted to her, as Somerville’s copy of another key work in late eighteenth century differential calculus, Joseph-Louis Lagrange’s *Théorie des Fonctions Analytiques...* (Lagrange, 1797), bears inscriptions of both her name and that of William Wallace.

Somerville was not blind to the importance of owning or having access to a mathematical library; in PR she reflected on the “long course of years in which [she] had persevered almost without hope” between first reading the “mysterious word Algebra” and finally acquiring what she described as the means to pursue her studies with “increased assiduity” (Somerville and Somerville, 1873, 80). Moreover, on her death her mathematical library was left to Girton College, so that her books could continue to benefit women interested in higher mathematics (Somerville and Somerville, 1873, 80).

It is unclear to what extent Somerville would have been able to engage with these texts in 1812. Early on in PR she briefly mentions that she studied French whilst living in London with her first husband between 1804 and 1807, and that when visiting Paris in 1817 she “was less at a loss on scientific subjects, because almost all [her] books on science were in French” (Somerville and Somerville, 1873, 109). However, later on she claims she felt “embarrassment and mortification... suffered from ignorance of the common European languages” which led her to engage language tutors for her daughters from a young age (Somerville and Somerville, 1873, 157). In addition, many of these texts were deemed too difficult even for highly trained mathematicians; for example, Playfair wrote in 1808 that “a man may be perfectly acquainted with everything on mathematical learning that has been written in this country [Great Britain], and may yet find himself stopped at the first page of the works of Euler or D’Alembert... from

want of knowing the principles and the methods which they take for granted as known to every mathematical reader”. Regarding Laplace’s *Mécanique Céleste* itself, Playfair estimated that no more than a dozen people in Great Britain could “read that work with any tolerable facility” (Playfair, 1808, 281). In order to investigate Somerville’s engagement with and understanding of the differential calculus, we now turn to her correspondence with William Wallace in 1816.

4. Using the Differential Calculus in Published Solutions

My correspondence with Mr Wallace began when he was Professor at the Military College at Marlow in consequence of problems given in the Mathematical Repository which I sometimes succeeded in solving & sometimes not. Mr Wallace sent his own solutions to me with criticisms on mine... I can never forget his kindness.

Note written by Somerville c.1870.
MSC, Dep. c. 372, Folder MSW-1.

Two letters written by William Wallace to Somerville in May of 1816, by which point she was now living in London with her second husband, further illuminate their relationship and Somerville’s contributions to MR.³⁵ In these letters Wallace offered criticism on work Somerville had previously shared with him, as well as enclosing further exercises on “the application of analysis to geometry”, claiming that “such exercises [are] useful to prepare for the study of analytical works”.³⁶ Wallace provided Somerville with his own solutions to the exercises that he set, sent concurrently in a sealed envelope, and for one exercise that Somerville had already completed sent her a solution “different, but not better” taken from the *Annales de Mathématiques pures et appliquées*.³⁷

Beyond providing materials for Somerville to use in her studies, William Wallace offered advice for developing good mathematical practice; he wrote “I hardly ever resolved a problem in the most direct manner possible at first: In general I find that a first solution may be improved and shortened, hence it always happens that a short and simple solution is the result of long meditation”. Wallace also strongly discouraged her from peeking at the solutions he

³⁵Along with a letter written by William Wallace in 1831, thanking Somerville for a copy of her *Mechanism of the Heavens*, and the aforementioned letter from Somerville to Wallace in 1812, these are the sole extant letters from the Somerville-William Wallace correspondence. MSC, Dep c. 372, Folder MSW-1.

³⁶MSC, Dep c. 372, Folder MSW-1, 12/05/1816 & 18/05/1816.

³⁷MSC, Dep c. 372, Folder MSW-1, 18/05/1816. Unfortunately the solutions mentioned in the letters are no longer extant. It seems that a volume of the *Annales de Mathématiques* had been seen by both William Wallace and his colleague Thomas Leybourn at the RMC, as solutions are also inserted in the next edition of the *Mathematical Repository*, namely Volume 4 in 1819 (see above). It is currently unclear whether Somerville herself would have had direct access to this journal in London.

sent before she had solved a question on her own. In these letters, Wallace kept Somerville updated on the ongoing ill health of his daughters, sharing his sorrow at the slow improvement of his eldest daughter, demonstrating that in the preceding four years their acquaintance had developed into friendship, as well as that of informal mentor and mentee.

As discussed earlier, three solutions by Somerville were included or mentioned in the third volume of MR, published in 1814, under the pseudonym “a Lady”. That Somerville was in fact behind all instances of this pseudonym is supported by a handwritten copy of the prize question (with solution) amongst the William Wallace letters in the Somerville papers, and copies of the remaining two solutions in a letter written by Somerville to Wallace in 1812. Three further solutions by “a Lady”, to Questions 377, 381, and 382 respectively (see Appendix B), were included in Volume 4, published in 1819. Wallace’s letters mention Somerville’s solution to one of these questions, Question 381 on the area of a lemniscata, as well as an attempted solution to Question 384 (see Appendix C and below). Unfortunately neither Wallace nor Somerville’s solutions are included with the letters; however, alternative copies of Somerville’s solutions can be found in one of her personal notebooks dating from the early 1820s.³⁸

Somerville’s notebook contains a series of scientific and mathematical investigations dated between 1821 and 1824, including a diagram of Encke’s comet³⁹ and investigations on the undulatory theory of light. Rather than making hasty jottings of ideas, Somerville here appears to have collected together neat summaries of both her own and others’ work. Copies of her solutions to seven questions contained in Volumes 3 and 4 of MR are the first entries of the notebook, along with four miscellaneous mathematical puzzles with solutions. These entries include all the solutions published by “a Lady”, except for Question 311, and solutions to three further questions included in Volume 4, namely questions 332, 384, and 387 (see Appendix C). These entries will be considered in conjunction with Wallace’s letters to analyse Somerville’s understanding of the calculus in 1816.

The first letter opens with Wallace’s feedback on Somerville’s attempted solution to “the 14th Question of the 14th No of the Mathematical Repository”, submitted by Paul Lawrence Baker⁴⁰ under the pseudonym ‘Palaba’:

XIV. QUESTION 384, *by* Palaba

³⁸MSC, Dep. c. 352, Folder MSSW-5.

³⁹In 1818 Johann Franz Encke calculated the orbit of a new comet, identifying it with observations made in 1786, 1795 and 1805. He was also able to predict the return of the comet, since designated ‘Encke’s Comet’, and it was indeed observed by Christian Rümker in Sydney, Australia on 2nd June 1822. In recognition of his work, described as “the greatest step that had been made in the astronomy of comets since the verification of Halley’s Comet in 1759”, Encke was awarded the Royal Astronomical Society’s Gold Medal in 1824 (Pritchard, 1866, 131).

⁴⁰I am grateful to Olivier Bruneau for this identification; unfortunately little biographical or mathematical information about Baker is known at this time.

Find the equation of the curve of which this is the property: if from a fixed point in the axis a perpendicular be drawn to it and produced to meet a tangent to any point in the curve, the length of this perpendicular and tangent together, shall be double the length of the curve between the vertex and the point from which the tangent was drawn.

Wallace begins by noting that Somerville has misused a formula given by himself in item 77 of his *Edinburgh Encyclopaedia* article entitled ‘Fluxions’ (Wallace, 1815, 424). It is interesting to note that Somerville had thus clearly read this article, which played a key role in the circulation of the calculus in Great Britain in the early 19th century. Indeed Guicciardini described it as “the first complete English treatise⁴¹ on the calculus written in differential notation” (Guicciardini, 1989, 120). Similarly to her 1812 solution to Question 324 discussed earlier, Somerville here demonstrates an awareness of and engagement with contemporary literature, but also conceptual misunderstandings of the mathematics in use. In his second letter, Wallace encloses his own solution to the question (no longer extant) and encourages Somerville to try again. He advises her to “avoid angular functions and to employ in [her] solution only the coordinates x , y and the arc z ”, and to replace $\frac{\partial y}{\partial x}$ with the symbol p for ease of calculation.⁴² In addition, Wallace gives Somerville a criterion that the curve should satisfy so that she may know when she has the correct solution: Let the curve meet the axis at A and C , where $3AB = BC$, then

$$3BC \times PQ^2 = BQ \times QC^2 \quad (1)$$

(see Figure 2).

The very first entry in Somerville’s notebook is a solution to Question 384 (see Figure 3), and it is clear that this solution was prepared after May 1816 as Somerville follows both of William Wallace’s suggestions above.⁴³ Using the notation from the diagram drawn by Somerville in her notebook solution (top of Figure 3), Question 384 asks for the equation of the curve BPG such that $AD + DP = 2BP$. Somerville begins her solution by letting $AB = a$, $BQ = x$, $PQ = y$, and $BP = z$. Here BP is the curve connecting B and P , such that $PC = dz$. Somerville investigates the lengths of AD and PD by constructing triangle PFC with side lengths $PF = dx$ and $FC = dy$. By similar triangles, $AD = y - \frac{(a+x)dy}{dx}$. By Pythagoras’ Theorem we have that $DP^2 = DE^2 + EP^2$ and $dz^2 = dx^2 + dy^2$. Hence $DP^2 = (a+x)^2(1 + \frac{dy^2}{dx^2}) \implies DP = (a+x)\frac{dz}{dx}$. Therefore $AD + DP = 2BP$ becomes $y - \frac{(a+x)dy}{dx} + \frac{(a+x)dz}{dx} = 2z$, the differential equation for the curve given.

⁴¹ Although published as an entry in the *Edinburgh Encyclopaedia*, the article was 86 pages long and thus easily warrants being described as a treatise!

⁴² MSC, Dep c. 372, Folder MSW-1, 18/05/1816.

⁴³ MSC, Dep. c. 352, Folder MSSW-5. Unfortunately the solution which Somerville originally sent to Wallace, and which prompted his critique, is not contained in the Somerville Collection in Oxford.

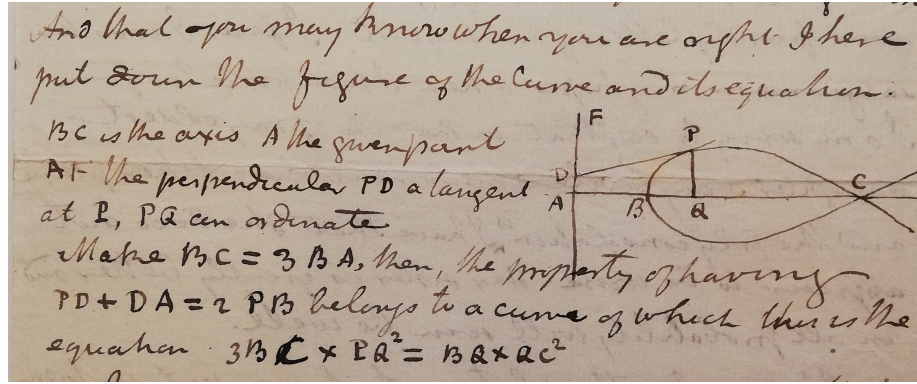


Figure 2: Extract of Willam Wallace's letter to Somerville, dated 18th May 1816, including his hint for answering Question 384, posed in Volume 3 of MR.

Following Wallace's hint and letting $p = \frac{dy}{dx}$ and $q = \frac{dz}{dx}$, Somerville takes differentials once and rearranges the equation into the form $\frac{dx}{a+x} = \frac{dq}{q} - \frac{dp}{q}$. As $q^2 = \frac{dz^2}{dx^2} = \frac{dx^2 + dy^2}{dx^2} = 1 + p^2$, this can be rewritten $\frac{dx}{a+x} = \frac{dq}{q} - \frac{dp}{\sqrt{1+p^2}}$ which integrates to

$$\log(a+x) = \log(\sqrt{1+p^2}) - \log(p + \sqrt{1+p^2}) + \log c$$

and after removing the logarithms we get

$$a+x = \frac{c\sqrt{1+p^2}}{p + \sqrt{1+p^2}}.$$

Somerville claims that as p is "infinite" when evaluated at the vertex B , where $x = 0$, then at this point "1 may be neglected in comparison of p ", giving

$$a+0 = \frac{c\sqrt{1+p^2}}{p + \sqrt{1+p^2}} = \frac{c\sqrt{p^2}}{p + \sqrt{p^2}} = \frac{c}{2}.$$

Therefore $c = 2a$ and

$$a+x = \frac{2a\sqrt{1+p^2}}{p + \sqrt{1+p^2}}.$$

Somerville rearranges to get $p = \frac{a-x}{2\sqrt{ax}}$, and using this result and that $p = \frac{dy}{dx}$, integrates to show that

$$y = \sqrt{ax} - \frac{x^{\frac{3}{2}}}{3\sqrt{a}}. \quad (2)$$

The curve cuts the axis in two places, at $x = 0$ (by construction) and $x = 3a$, which Somerville labels G (see Figure 3). Somerville proceeds to check that the

curve she has found satisfies the criterion given by Wallace. She begins by squaring and rearranging the equation of the curve, equation (2), to get

$$9ay^2 = x(3a - x)^2,$$

which she re-writes as

$$9a : x :: (3a - x)^2 : y^2,$$

and then gives in terms of line segments as⁴⁴

$$3BG : BQ :: QG^2 : PQ^2.$$

Somerville concludes her solution here, but we can see by rearranging and relabelling this expression that the curve does indeed satisfy equation (1);

$$3BC \times PQ^2 = BQ \times QC^2.$$

Notably, Somerville here uses the notation and terminology often described by British mathematicians at the beginning of the nineteenth century as ‘foreign’ or ‘continental’; she uses dx rather than \dot{x} , and speaks of ‘integrating’ rather than ‘taking the fluents’. This is especially significant when compared with the two solutions to this question actually published in MR. The first was given by ‘Palaba’, the proposer, and the second by William Wallace himself. Baker’s solution used similar triangles and fluxional calculus, whereas Wallace used differential notation, dx , dy , etc. However, Wallace’s terminology does not match his notation; rather than ‘differentiating’ he ‘takes the fluxions’, and later he ‘takes the fluents’ when applying the inverse process. This seeming mismatch between notation and language is consistent throughout Wallace’s other solutions in MR and is also witnessed in occasional solutions contributed by Messrs Lowry and Cunliffe, both professors at the Royal Military College with William and later John Wallace.⁴⁵ Other solutions which utilised the calculus in Volume 4 of MR utilised fluxional notation, such as \dot{x} , intermingled with the elongated s symbol of integration. Out of the 80 questions published in this volume, 27 had at least one solution which utilised calculus of some sort; of those solutions, 10 used purely differential notation and language. Seven of those were submitted by John Herschel, another member of the Analytical Society in Cambridge, one by William Wallace, and two were submissions by Mary Somerville. Therefore, considering the ‘revolutionary spirit’ with which differential notation was employed by those who wished to see the adoption of continental methods in British mathematics, Somerville is here both clearly

⁴⁴The ratio notation here means the ratio of $3BG$ to BQ is equivalent to the ratio of QG^2 to PQ^2 ; or more succinctly $\frac{3BG}{BQ} = \frac{QG^2}{PQ^2}$.

⁴⁵For example in his solution to question 279 which “determine[s] the nature of the curve which touches an infinite number of lines of a given kind, described upon a plane according to some determinate law”, and to question 358, which treats the sum of an infinite series (Leybourn, 1806–1835, 65, Vol 3), (Leybourn, 1806–1835, 54, Vol 4).

identifying herself with the mathematical practice of this reform community, and contributing significantly to its visibility.

Beyond a mere commitment to differential notation, the two solutions belonging to Somerville which utilised the calculus and were printed in Volume 4 also demonstrate Somerville's expanding mathematical skill-set. Both solutions, to Questions 381 and 382 respectively, use trigonometrical functions and the differential calculus to investigate the properties of analytical curves; namely, curves described by a formula. An early solution to each question can be found in her notebook, and we proceed now to compare the published and unpublished solutions to Question 381, in order to display progress in Somerville's mathematical aptitude.

XI. QUESTION 381, *by* Palaba.

The equation to the lemniscata being $(x^2 + y^2)^2 = x^2 - y^2$; find its area contained between the values of $x = 0$ and $x = 1$.

Both the published solution, found in Volume 4 of MR, and the unpublished solution, being the fourth undated entry in the notebook, begin in the same manner (Leybourn, 1806–1835, 95, Vol 4). Somerville lets CPA be the lemniscata under consideration, where C is the 'centre', CA is the semi-axis,⁴⁶ and P is a point on the top right-hand side of the curve (see Figure 4). She then introduces the polar coordinates r and ϕ , where r is the 'variable radius' CP , and ϕ is the 'variable angle' PCA . Hence $x = r \cos \phi$ and $y = r \sin \phi$, and the equation to the curve becomes

$$r = \sqrt{\cos^2 \phi - \sin^2 \phi}.$$

Somerville then finds y and dx in terms of ϕ , and substitutes them into the 'general expression for areas', which she gives as $\int y dx$. This gives

$$\int y dx = \int d\phi \sin^4 \phi - 3 \int d\phi \sin^2 \phi \cos^2 \phi$$

and it is here that the solutions diverge.

First we consider the unpublished solution. Here Somerville computes the integration term by term, giving (with a missing three inserted into the left-hand side of the second line)

$$\int d\phi \sin^4 \phi = -\frac{\cos \phi \sin^3 \phi}{4} + \frac{3}{4} \int d\phi \sin^2 \phi$$

and

⁴⁶A minor difference between the two solutions, is that in the unpublished version Somerville takes a as the length of the semi-axis, rather than 1, which is carried through the solution; it is silently amended here for ease.

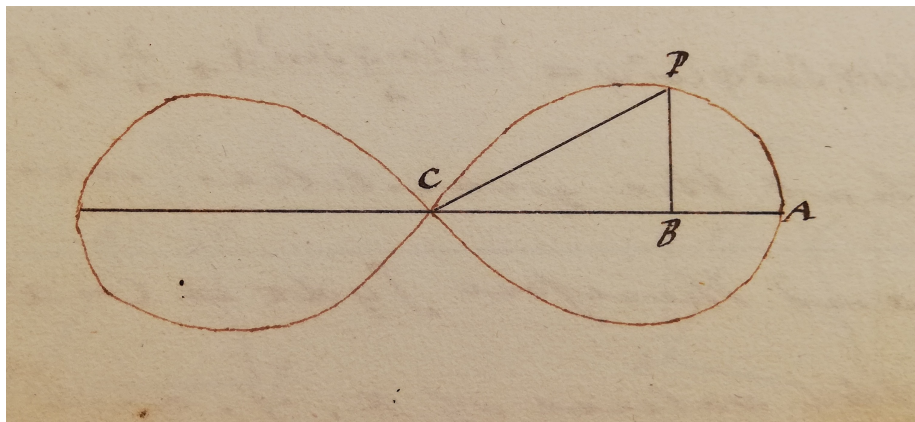


Figure 4: The diagram used by Somerville in her notebook answer to Question 381.

$$3 \int d\phi \sin^2 \phi \cos^2 \phi = \frac{3 \cos \phi \sin^3 \phi}{4} + \frac{3}{4} \int d\phi \sin^2 \phi.$$

On subtracting the latter from the former, the terms under the integral sign cancel out, giving $\int y dx = c - \cos \phi \sin^3 \phi$. Unfortunately, it is unclear how Somerville computed these integrals, as she has omitted all of her working. Somerville continues her solution by putting the equation for the area in terms of x and y ,

$$\int y dx = c + \frac{xy^3}{y^2 - x^2},$$

and concludes by subtracting the value of this expression at $x = 1$ from its value at $x = 0$, to give the area as $\frac{y^3}{y^2 - 1}$.

This answer is clearly in the wrong form, as the solution should not be dependent on y ; y is a function of x , and Somerville is integrating on an interval where the function is well defined, so the result of the integration should give a constant. However, perhaps because the function is given implicitly, and moreover is multivalued at the limits $x = 0$ and $x = 1$, Somerville was not able to evaluate the result when written in this form. This difficulty is overcome in the solution published in MR, as she instead gives the value of the integral in terms of the ‘variable radius’ r ,

$$\text{area} = \int y dx = c - \frac{1}{4}(1 - r^2)\sqrt{1 - r^4}.$$

She then evaluates this expression at $r = 0$ ($x = 0$), and $r = 1$ ($x = 1$), and subtracts the latter from the former to give the area of one half oval as $\frac{1}{4}$ (and implicitly, by symmetry, the total area under the curve as equal to 1).

Therefore we see Somerville’s fluency with trigonometric functions and polar coordinates increase between her first and second solution. In addition, she

demonstrates an improving fluency in methods of integration in her calculation of

$$\int y dx = \int d\phi \sin^4 \phi - 3 \int d\phi \sin^2 \phi \cos^2 \phi$$

in the published solution. Rather than computing the entire integral term by term, Somerville instead calculates

$$\int d\phi \sin^4 \phi = \int (d\phi \sin \phi) \sin^3 \phi = -\cos \phi \sin^3 \phi + 3 \int d\phi \cos^2 \phi \sin^2 \phi,$$

and notes that second term of the result cancels out the second term of the expression to be integrated. Her presentation of this integral strongly suggests that Somerville here used the ubiquitous method now commonly known as Integration by Parts. For completeness, we note that the other solution to this question published in MR, which was submitted by the proposer ‘Palaba’, used fluxional calculus to reach the answer.

5. Becoming Known as a Mathematician

Mr La Place said with regard to the Mech. Cel. it is probable that improvements may be made in analysis and that methods may be found to make the series converge whither the inclination of the planes of the orbits of the planets be great or small...

Diary entry by Mary Somerville, Paris, 6/08/1817.
MSC, Dep. c. 355, Folder MSAU-1.

Although her publications in the *Mathematical Repository* were published anonymously, under her pseudonym “a Lady”, by the mid-1820s Mary Somerville had nevertheless built a reputation across western Europe for having read and understood advanced and esoteric mathematics.

We first see Somerville described as a mathematician in 1812, in letters written to and by the eminent astronomer William Herschel, who was well known for his discovery of the planet Uranus in 1781 (Hoskin, 2008). On her marriage in 1812 to Dr. William Somerville⁴⁷ the newly-weds briefly moved away from Edinburgh to the south coast of England. During their long journey southwards, they paid a visit to Somerville’s mentor William Wallace in Marlow. In advance of their visit, Wallace asked Herschel if he could call on him in Slough accompanied by the Somervilles; Herschel replied that he would be “very happy to see the Lady... and you may be assured that the trait in the character of a Lady to be a good mathematician without Wrangleship⁴⁸ will be highly

⁴⁷I will continue to refer to Somerville as such, and distinguish her husband as Dr. Somerville.

⁴⁸*Wrangleship* here refers to the title of ‘Wrangler’ which was given to those who achieved the highest marks on the Senate House, or Tripos Exam at Cambridge University (Craik, 2007, 3), see footnote 16.

esteemed”.⁴⁹

John Playfair also furnished the Somervilles with a letter of introduction addressed to William Herschel. In this letter Playfair claimed that “Mrs Somerville is distinguished by knowledge of the Mathematical Sciences rarely to be met with in men. She has studied Geometry and algebra with great success, & is particularly well acquainted with astronomy”.⁵⁰ Playfair’s favourable comparison between Somerville and her male contemporaries is certainly a double-edged sword. As an author well known for his views on the decline of British mathematics and advocacy for the further study of continental methods in Britain, see (Ackerberg-Hastings, 2008), it is not unexpected that Playfair would be interested in a mathematician successfully pursuing this line of study. However, his claim that a *woman* had overtaken most mathematicians in Britain, at a time when women were not expected or encouraged in intellectual pursuits, legitimises Somerville whilst being purposefully inflammatory. This can be seen more explicitly in Robert Mackenzie Beverley’s 1833 letter on the corrupt state of the University of Cambridge to that institution’s Chancellor, where he claimed “that the most eminent mathematician of England is at this present time a lady! Mrs. Somerville has passed by the flaming walls of Cambridge, and rising like the ethereal sun, has dimmed all the college stars into pale obscurity; and so it is, that the discoveries in science for the last twenty years have not been made or even suggested by the sages of Cambridge” (Beverley, 1833, 39).

Nevertheless, Playfair’s letter supports Somerville’s account given in her Personal Recollections of having discussed with him her difficulties on reading *Mécanique Céleste*, and moreover suggests that by 1812 she was beginning to overcome these difficulties and seriously pursue mathematical learning. The Somervilles’ acquaintance with William and his wife Mary Pitt Herschel lasted throughout their lives. At this time Somerville also met William’s son John Herschel, then a student at Cambridge University and later a fellow contributor to the *Mathematical Repository*, with whom she would correspond on scientific matters for most of her life, and intensely so during the preparation of her translation of Laplace’s *Mécanique Céleste* published as *Mechanism of the Heavens* in 1831.⁵¹ Somerville claims in PR that William Herschel’s sister Caroline Herschel, also renowned for her astronomical observations, was abroad at the time of this visit, and makes no mention of having met her on a subsequent occasion (Somerville and Somerville, 1873, 106).⁵²

The Somervilles returned to live in Edinburgh in 1813, where in early 1816 they became acquainted with Leonard Horner, an active member of the Geolog-

⁴⁹MSC, Dep c. 370, Folder MSH-4, 8/07/1812.

⁵⁰Royal Society, Herschel Papers, HS 14.169, 16/06/1812.

⁵¹See forthcoming PhD thesis by this author.

⁵²However, Caroline Herschel’s memoir suggests the two women were aware of each other’s astronomical work, and in 1835 Somerville wrote to Herschel on the occasion of them being simultaneously elected the first women to be honorary members of the Royal Astronomical Society, and offered a copy of her second book *On the Connexion of the Physical Sciences* (Somerville, 1835) (Herschel, 1876, 274).

ical Society and later an influential factory inspector (Bartrip, 2008).⁵³ Horner was evidently impressed by Somerville, as on her move to London a few months later he sent a letter to his friend Alexander Marcet in which he described her as “a person of very extraordinary acquirements, particularly in mathematics. But she has not a shade of blue in her stockings” (Patterson, 1983, 12).⁵⁴ Horner furthermore asked that Jane Marcet, well-known author of the highly successful 1806 *Conversations on Chemistry* and wife of Alexander, pay a call to the Somervilles on their arrival in London. Jane Marcet agreed, and the two scientific women became lifelong friends.

The Marcets introduced the Somervilles into a thriving metropolitan community which included fellows of the Royal Society, the Linnean Society, and the Geological Society; all three of which William Somerville himself became a fellow or member of by 1817. Together, the Somervilles frequently attended lectures at the Royal Institution on Albemarle Street, where both Humphry Davy and Michael Faraday presented their formative research in chemistry. PR is filled with anecdotes from Somerville’s time in London which feature notable natural philosophers, such as making astronomical observations in little gardens with physicist Henry Kater, and light experiments using prisms and crystals with chemist and physicist William Hyde Wollaston.

The Marcets were at the centre of a distinctly European social set, and during gatherings at their house Somerville became acquainted with Joseph-Louis Gay-Lussac, François Arago and Jean-Baptiste Biot, all of whom visited London in 1816–7 for their research.⁵⁵ After his departure to Edinburgh, Biot wrote to Dr Somerville to thank him for the letters of introduction that he had provided him with. In this letter of 1 June 1813 Biot requests permission to write directly to Somerville herself, and begs her to contact him with any difficulty she may meet in mathematics; thus Biot was clearly aware of her studies.⁵⁶ Over four years later Biot finally took advantage of his permission to write to Somerville personally, in order to encourage her to visit him in Paris, where he promised a warm welcome from both him and his wife, and the existence of a group of people who already had a great desire to make her acquaintance.⁵⁷ Somerville was clearly convinced, as on 17th July 1817 she began the five day journey to

⁵³Horner’s brother Francis was a prolific contributor to the *Edinburgh Review*, and authored an anonymous English translation of Euler’s *Algebra* in 1797 (Thorne, 2005).

⁵⁴Originating from the eighteenth-century Bluestockings Society led by Elizabeth Montagu, in the nineteenth century ‘bluestocking’ became a derogatory term for a woman interested only in intellectual pursuits (Griffin, 2017).

⁵⁵Not unusually for the time, Somerville describes Gay-Lussac, Arago and Biot as ‘philosophers’ in PR, as well as Simeon-Denis Poisson who she subsequently met in Paris (Somerville and Somerville, 1873, 110).

⁵⁶“Je l’ai priée, si elle rencontrait quelques difficultés dans les études mathématiques de vouloir bien me les envoyer et je ne lui ferai pas attendre la réponse”, MSC, Dep. c. 369, Folder MSB–8, 01/06/1813.

⁵⁷“Pour vous madame vous allez à Paris ; et vous y trouverez je vous assure des personnes qui ont déjà une très grande envie de vous voir”, MSC, Dep. c. 369, Folder MSB–8, 01/06/1817, as referenced in (Patterson, 1983, 17).

Paris accompanied by her husband and brother, where they were to stay for two weeks en route to Geneva and Rome.

Whilst in Paris, Somerville kept a diary where she detailed her visits to the Institut de France, accompanied by Gay-Lussac, Arago and Madame Biot, to the Jardin des Plantes where she was hosted by botanist Georges Cuvier, and to l'École des mines with geologist André Brochant de Villiers. She visited the Paris observatory multiple times where she saw Claude Louis Mathieu, an astronomer and member of the prestigious Académie des Sciences and Bureau des Longitudes. Somerville writes that she “received the greatest attention” from Madame Gabrielle Biot, who herself had authored a mathematical translation of Ernst Gottfried Fischer’s *Lehrbuch der Mechanischen Naturlehre* in 1806 (Patterson, 1983, 21). On the 30th of July Madame Biot hosted a dinner party “on purpose to show [Somerville], as she said, les personnes distinguées [sic]”; in attendance were the Aragos, the Gay-Lussacs, Alexander von Humboldt, and Siméon-Denis Poisson.⁵⁸ Unfortunately, we learn little about what was discussed at this dinner party, as two pages are missing from the diary!

A few days later on the 6th of August, the Somervilles dined at Arcueil, hosted by Pierre-Simon Laplace, with whom Mary Somerville discussed potential improvements to the mathematical analysis which underpins his *Mécanique Céleste* (see quote at the beginning of this section). According to her diary, on complimenting Laplace on his *Système du Monde* for its “depth of science and elegance of composition”, he replied “it was very true but he did not think the English or any other nation could appreciate the beauties of French literature.”⁵⁹

After leaving Paris, when her diary tails off, the Somervilles continued on to Geneva, before spending the winter and spring in multiple cities across Italy and returning to London in late summer of 1818.⁶⁰ Somerville returned to continental Europe in 1824 when, along with her husband and eldest son from her first marriage, she visited what is now Belgium, the Netherlands and Germany. In Brussels she became acquainted with astronomer Adolphe Quetelet, who would later publish translations and reviews of Somerville’s work in a journal he edited,⁶¹ namely the *Correspondance Mathématique et Physique*; in Bonn the Somervilles renewed their acquaintance with Alexander von Humboldt; and whilst in Utrecht Somerville met astronomer Gerard Moll (Patterson, 1983, 45).⁶²

⁵⁸MSC, Dep. c. 355, Folder MSAU–1, 30/07/1817.

⁵⁹MSC, Dep. c. 355, Folder MSAU–1, 06/08/1817.

⁶⁰Although they appear to have made a good impression on polite society, there is no evidence of Somerville actively pursuing her mathematical interests, nor making the acquaintance of others who could be named as ‘mathematicians’ (Patterson, 1983, 26–30).

⁶¹see MSC, Dep. c. 372, Folder MSQ–1, 27/09/1827 and MSC, Dep. c. 372, Folder MSQ–1, 14/03/1832.

⁶²Moll would later assist Somerville in her preparation of an article on comets for the *Quarterly Review* (Somerville and Secord, 2004, I.7, Vol 1) (Patterson, 1983, 166–7), and himself published a rebuttal to Charles Babbage’s 1830 polemic *Reflections on the Decline of Science in England* (Babbage, 1830) (Moll, 1831) (Reingold, 1968).

Somerville continued by letter many of the acquaintances made whilst travelling Europe. Most notably she received two letters from Pierre-Simon Laplace, the first of which was written in 1824. His negative opinions of the English seem to have changed in the preceding seven years, and it is clear that Somerville herself had impressed him with her mathematical knowledge. He wrote:

The interest that you deign to take in my works is especially flattering, because they have so few similar readers nor judges so enlightened... I see with great pleasure your [British] mathematicians now engaging in analysis; and I do not doubt that in following this method with the sagacity proper to your nations, they will be led to important discoveries... Allow me, Madame, to offer you the attached copy of the fifth edition of my *Exposition du Système du Monde*, as a sign of my lively and respectful recognition. I would have liked to express it in person in London with Monsieur Magendie, as I had initially intended, but diverse occupations prevented me from accompanying my learned colleague, who has highly praised your welcome, and that of your scientists, as a welcome worthy of their scientific work and character.⁶³

Therefore by the mid-1820s, although she remained excluded from scientific societies and institutions, and her only mathematical publications were anonymous, she had not only studied advanced mathematics but had built a reputation for herself as a competent reader of mathematical analysis. She was especially known for her understanding of the mathematics of Pierre-Simon Laplace, which was described by Playfair as the most important work to distinguish the conclusion of the eighteenth, and the commencement of the nineteenth century (Playfair, 1808, 250). Indeed, her subsequent ‘career’ as an author began in 1827 with a letter from Henry Brougham (see section 1) to William Somerville, claiming that “unless Mrs Somerville will undertake [a translation of Laplace’s *Mécanique Céleste*] — none else can” (Somerville and Somerville, 1873, 161–2).

⁶³ “L’intérêt que vous daignez prendre à mes ouvrages me flatte d’autant plus, qu’ils ont bien peu de semblables lecteurs et de juges aussi éclairés... Je vois avec un grand Plaisir vos mathématiciens se livrer maintenant à l’analyse; et je ne doute point qu’en suivant cette méthode avec la sagacité propre à votre nation, ils ne soient conduits à d’importantes découvertes... Permettez-moi, Madame, de vous offrir l’exemplaire ci-joint, de la cinquième édition de mon *Exposition du Système du Monde*, comme un hommage de ma vive et respectueuse reconnaissance. J’aurais bien désiré vous l’exprimer de vive voix en allant à Londres avec Monsieur Magendie, comme je l’avais d’abord projeté, mais diverses occupations m’ont empêché d’accompagner mon savant confrère qui se loue extrêmement de votre accueil, et de celui de vos savants, accueil dont il est digne par ses travaux scientifiques et par son caractère”, 15th August 1824, (Hahn, 2013, 1250–1).

Conclusion

In conclusion, Somerville's contribution to the reformation of British mathematics began much sooner than previously appreciated. Indeed, twenty years before *Mechanism of the Heavens*, for which she has previously been recognised, Somerville was already an active participant in a community of reformers through her submissions to the *New Series of the Mathematical Repository*. Although Somerville's contributions to MR were published under a pseudonym, she was still able to build a reputation as an accomplished mathematician and gain recognition from the scientific community through her engagement in polite society throughout Great Britain, and during her time in Paris.

The usage of differential notation as a 'revolutionary' act was recognised at the time by practitioners such as William Wallace and James Ivory, and has since been recognised by historians of mathematics; Somerville's submissions significantly increased the presence of this notation in MR, a key node of reform. Moreover, her correspondence with William Wallace and early drafts of her solutions provide new insight into her mathematical formation, specifically in the differential calculus. This in turn illuminates the accessibility of this information in Great Britain at the beginning of the nineteenth century.

Appendices

Appendix A. *Mathematical Repository* Questions solved by 'a Lady' in Volume 3

All solutions published under the pseudonym 'a Lady', in Volumes 3 and 4 of MR, can be found in Volume 1 of Somerville's Collected Works, edited by James Secord (Somerville and Secord, 2004, 1, Part 1, I.1–I.5).

XX. PRIZE QUESTION 310, *by* Mr. W. Wallace.

Find such integer values of x, y, z as shall render the three expressions $x^2 + axy + y^2$, $x^2 + a'xz + z^2$, $y^2 + a''yz + z^2$ squares, a, a', a'' being given numbers.

First solution, by a Lady. Second solution, by Mr. Lowry.

I. QUESTION 311, *by* Mr. John Hynes, *Dublin*.

To divide a given square number n^2 , into two such parts that the sum of their squares and the sum of their cubes may both be rational squares.

First solution, by a Lady. Second solution, by Mr. Cunliffe, R. M. College. Third solution, by Mr. Lowry, R. M. College.

XIV. QUESTION 317, *by* G. V.⁶⁴

⁶⁴G. V. was a pseudonym of William Wallace (Craik, 1999, 245).

Let $ABCD$ be a parallelogram, draw the diagonal BC , and draw DE perpendicular to BC ; then, perpendiculars drawn to AB , AC at the points B and C shall intersect each other in the line DE .
Required the demonstration?

First solution, by Mr. John Dawes, Birmingham. Second solution, by Eratosthenes. Ingenious demonstrations were received from Messrs. Adams, Baines, and a Lady.

Appendix B. *Mathematical Repository* Questions solved by ‘a Lady’ in Volume 4

VII. QUESTION 377, *by* Mr. Cunliffe.

What is the relation of the diameters of the three circles, passing through the extremities of the sides, and point of intersection of the perpendiculars from the angles upon the sides of a plane triangle?

First solution, by a Lady. Second solution, by Mr. Cunliffe, the Proposer.

XI. QUESTION 381, *by* Palaba.

The equation to the lemniscata being $(x^2 + y^2)^2 = x^2 - y^2$; find its area contained between the values of $x = 0$ and $= 1$.

First solution, by a Lady. Second solution, by Palaba, the Proposer.

XII. QUESTION 382, *by* Palaba.

Determine that point in a curve whose equation is $a^{n-1}x = y^n$ to which a line must be drawn from the vertex making the greatest angle with the curve.

First solution, by a Lady. Second solution, by Palaba, the Proposer.

Appendix C. Remaining *Mathematical Repository* Questions solved by Mary Somerville in her Notebook

The following questions were all published in Volume 4 of the *New Series of the Mathematical Repository*.

II. QUESTION 332, *by* Mr John Hynes.

To find two fractions such that the sum and sum of their squares shall both be rational squares; and either of them being added to the square of the other shall make the same square.

Solution by Mr Cunliffe.

XIV. QUESTION 384, *by* Palaba

Find the equation of the curve of which this is the property: if from a fixed point in the axis a perpendicular be drawn to it and produced to meet a tangent to any point in the curve, the length of this perpendicular and tangent together, shall be double the length of the curve between the vertex and the point from which the tangent was drawn.

First Solution, by Palaba, the Proposer. Second solution, by Mr. W. Wallace, R. M. College.

XVII. QUESTION 387, *by* Palaba

TB , BC are the subtangent and ordinate of a curve whose vertex is A , and the tangent of the angle TCA is the tangent of the angle ACB in a given ratio. What is the nature of the curve?

Solution by Palaba, the Proposer.

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